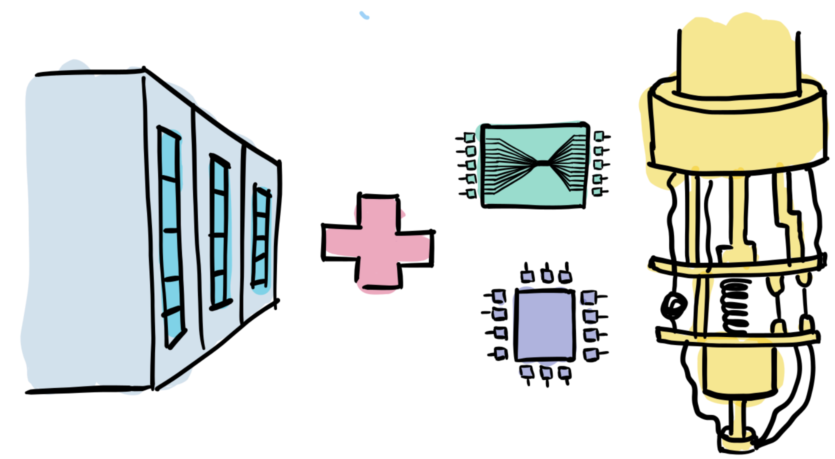
CALCOLO QUANTISTICO PER ALGORITMI VARIAZIONALI

Davide Vodola

28/04/2021

AlmaHAI: Hard Sciences
KICK-OFF WORKSHOP

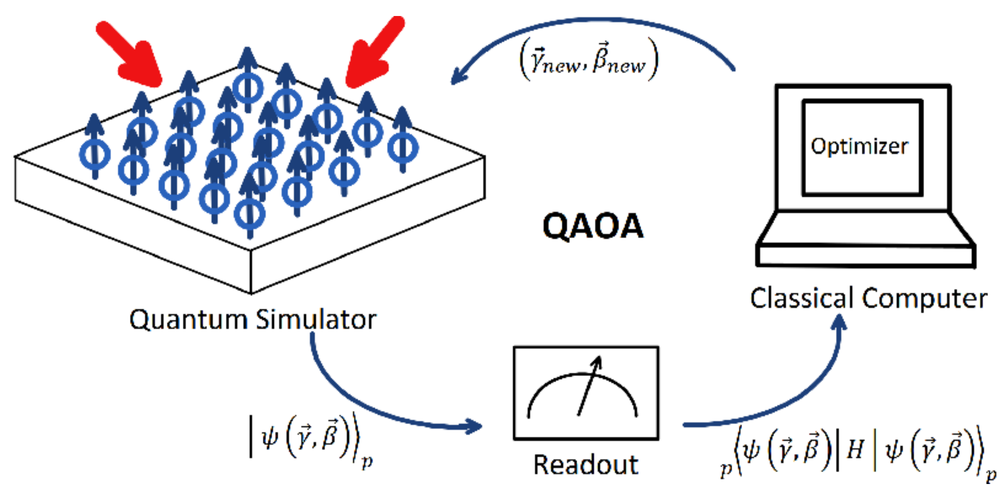
Outline



- What are quantum computers?

- What types of problem can be solved on a quantum computer?

- Variational algorithms for optimization problems



A quantum computer is ...

a computer
which works based on the laws of
quantum physics

Central ingredients:

- quantum **superposition** principle
- **entanglement**

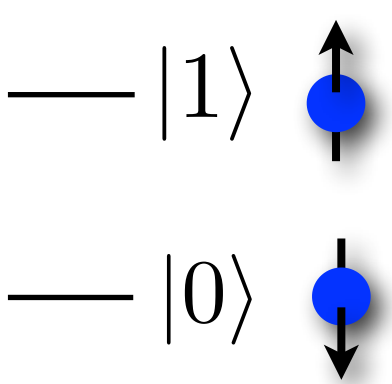
Basic unit in **classical** information: the bit

0

or

1

Basic unit in **quantum** information: two-level system = quantum bit (**qubit**)



$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle$$

with $c_0, c_1 \in \mathbb{C}$

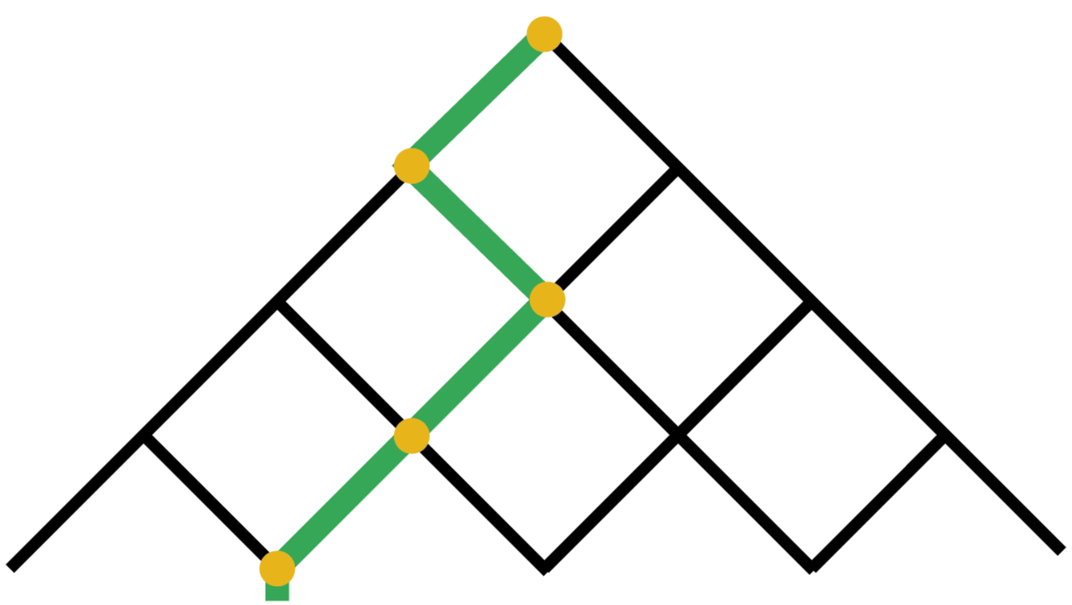
$$p_0 = |c_0|^2,\ p_1 = |c_1|^2$$

Superposition and entanglement

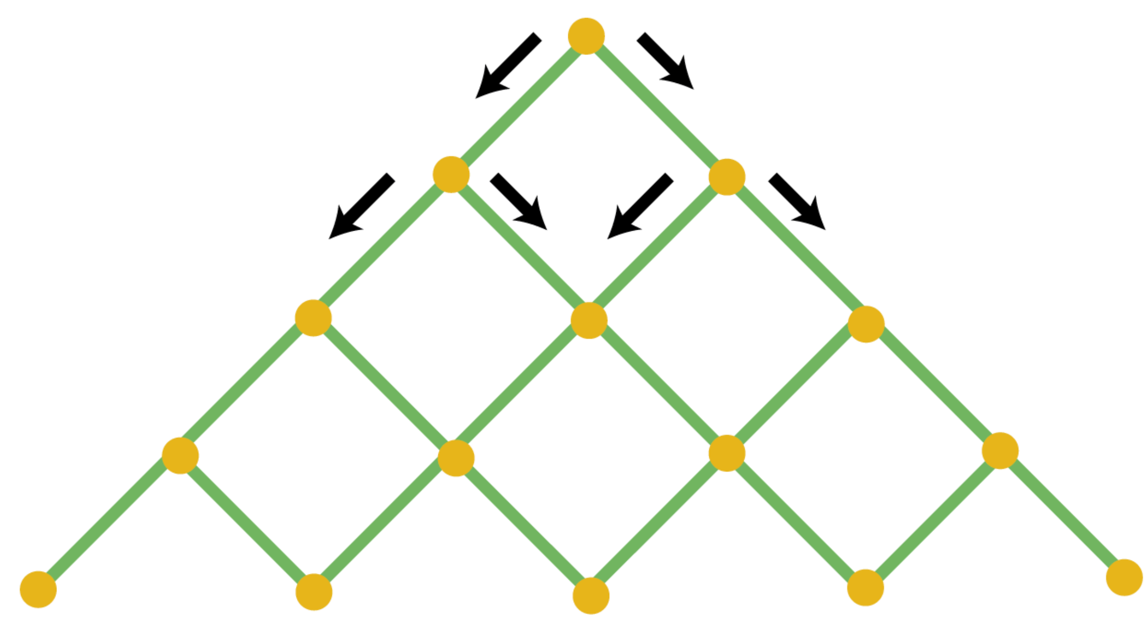
- A classical computer moves one step at the time at returns one answer

- A quantum computer can exploit the superposition principle and operates in parallel

Classical



Quantum



Superposition and entanglement

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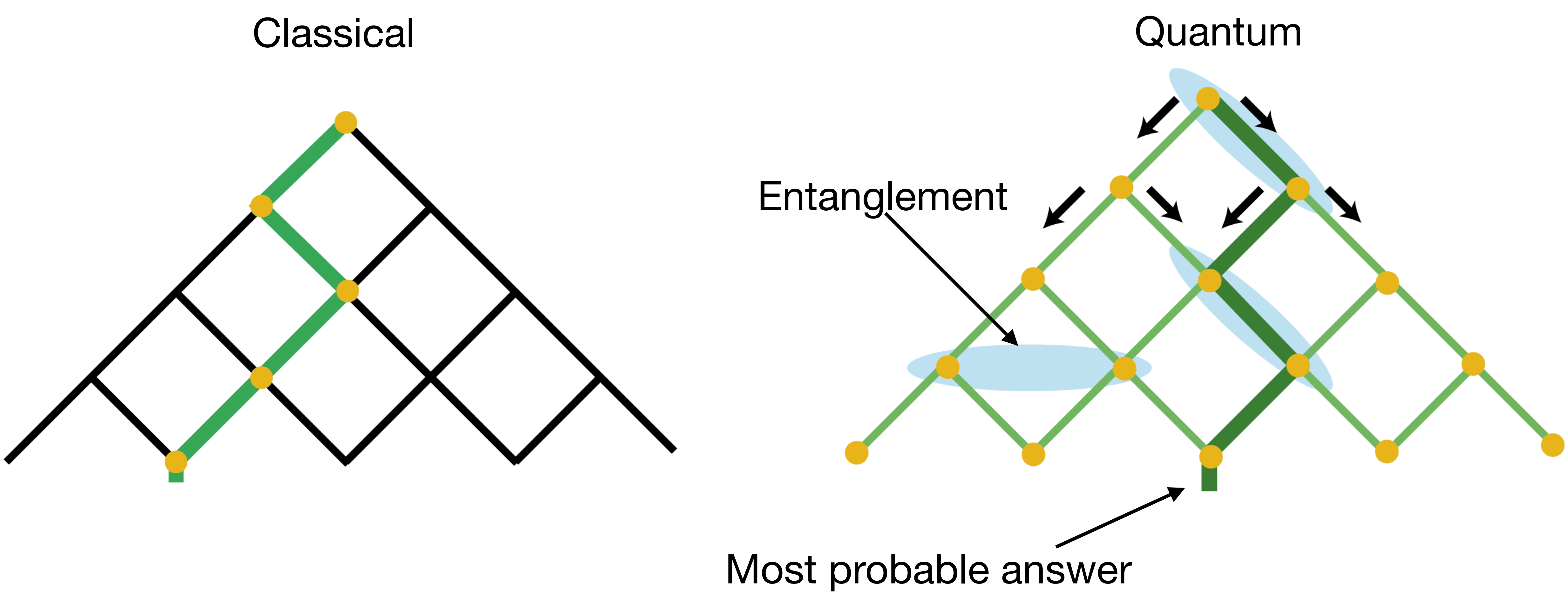


Quantum algorithms map the initial state into the final superposition containing the correct answer (with high probability)

What physical platforms can be quantum processors?

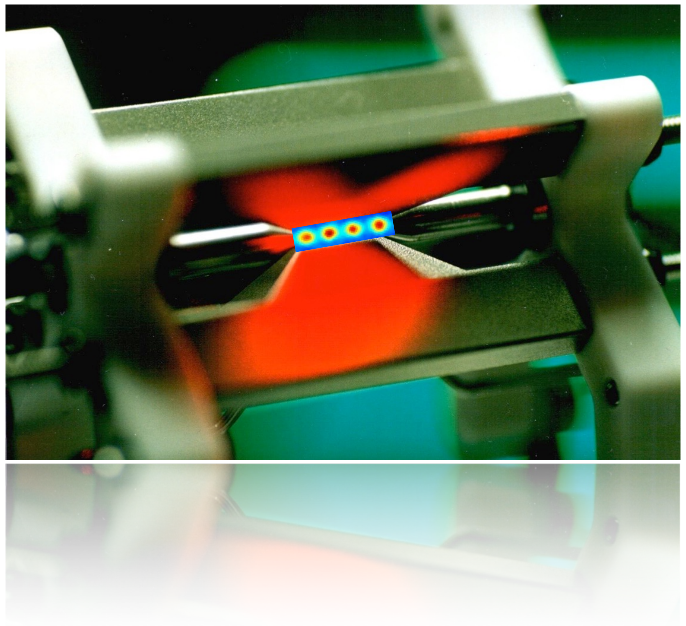
A quantum processes a physical platform needs to satisfy:

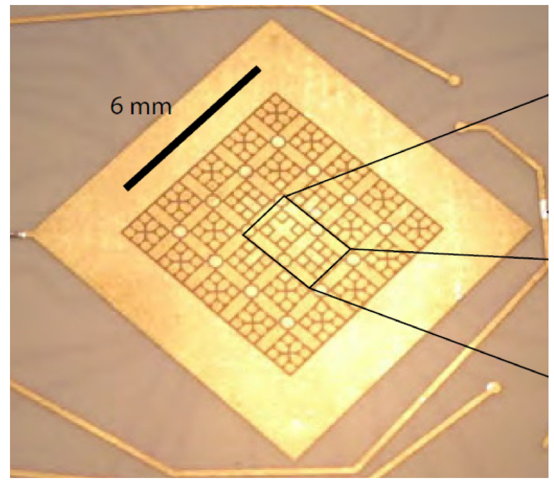
- Scalable system of well-characterized qubits
- Ability to initialize to a fiducial state
- Long coherence time (for low error rate)
- Universal set of quantum gates
- Capable of measuring any specific qubit

DiVincenzo criteria (2000)

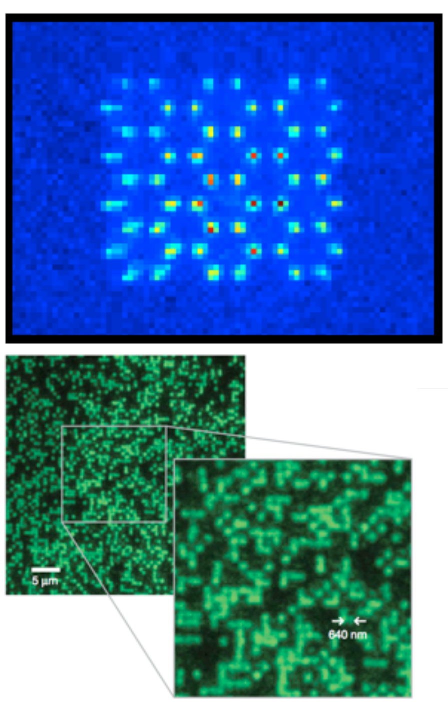
Which physical platform?

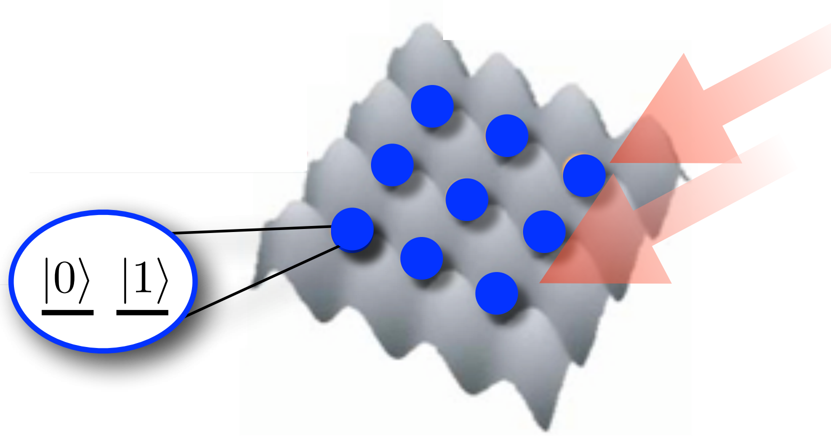
- From 1D to 2D **Ion Traps**



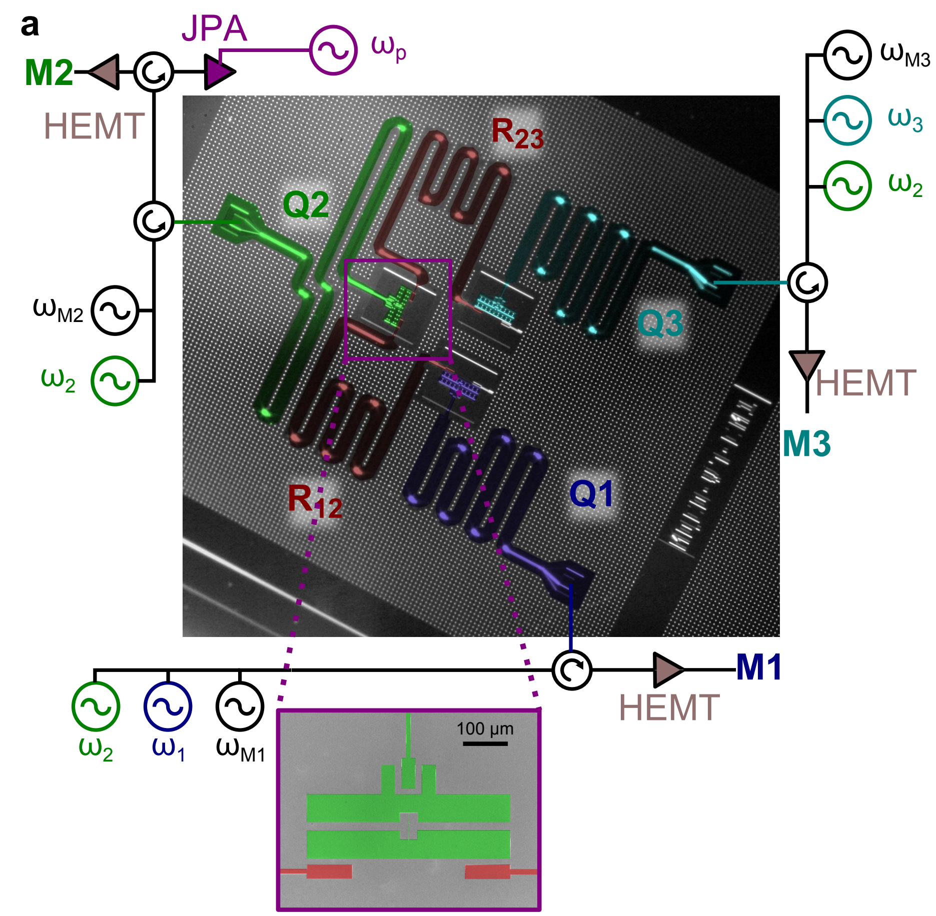


- **Cold Rydberg atoms**





- super-conducting qubits

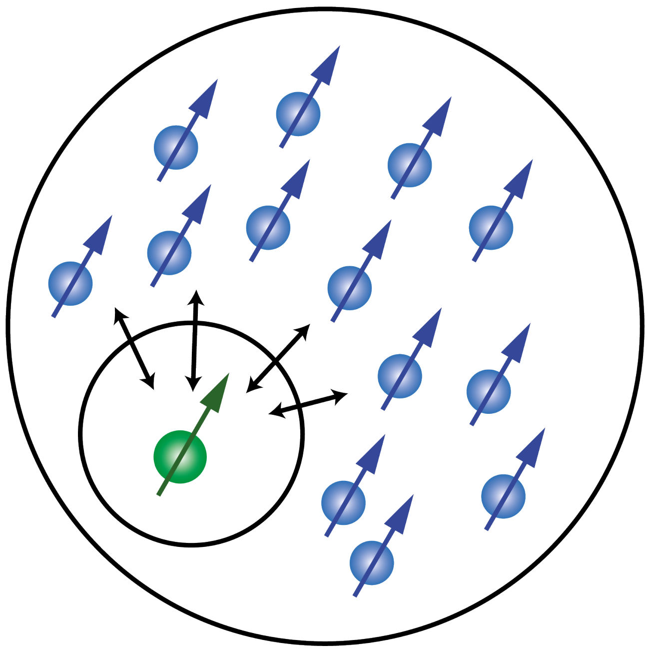


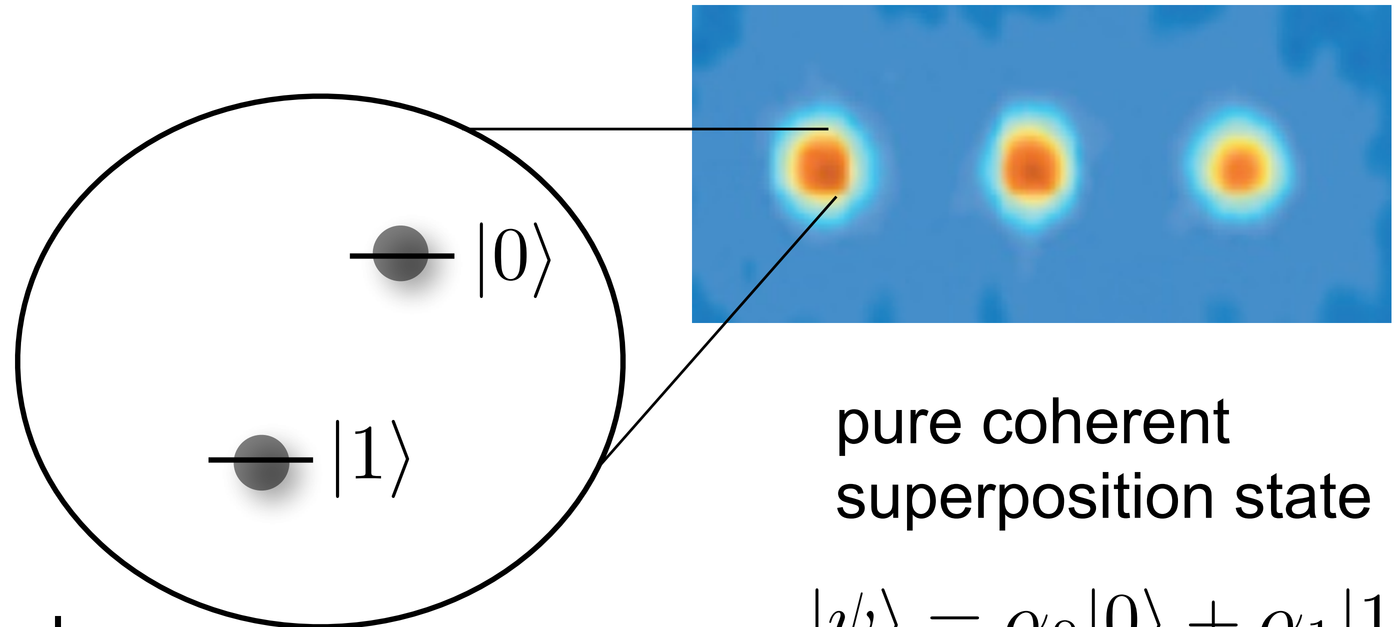
- Photons
- NV centres
- Quantum Dots

…

Main obstacle towards quantum computers: decoherence & errors

- Coupling to the environment causes decoherence:





example:
magnetic field fluctuations

pure coherent superposition state

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

dephasing

$$\rho = |\alpha_0|^2|0\rangle\langle 0| + |\alpha_1|^2|1\rangle\langle 1|$$

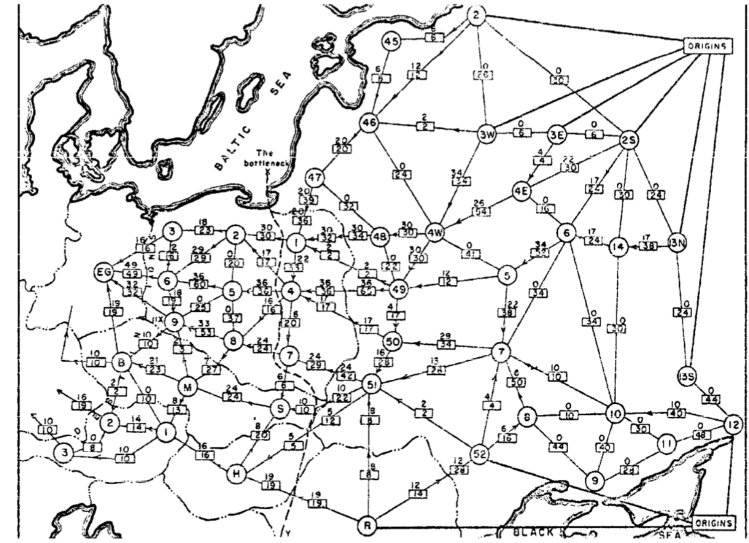
classical random bit

A long-term goal is to implement error detection and correction procedures

Meanwhile we should use current
Noisy Intermediate Scale Quantum (NISQ) devices

Preskill, Quantum Computing in the NISQ era and beyond, Quantum (2018)

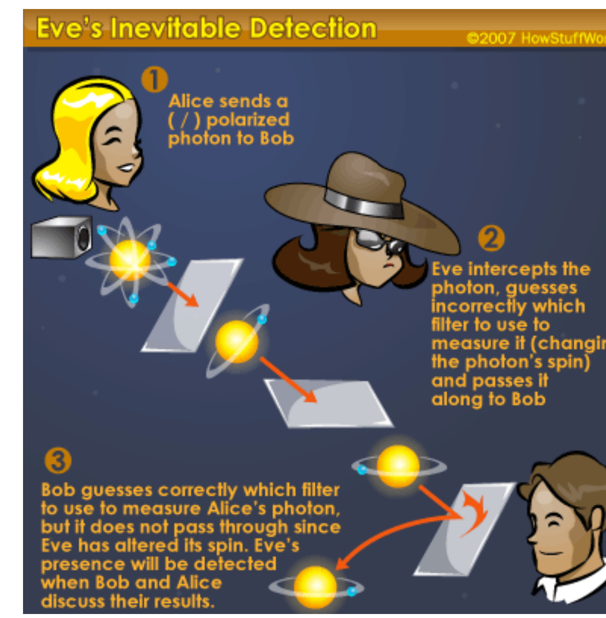
Which problems can be solved by a quantum computer?

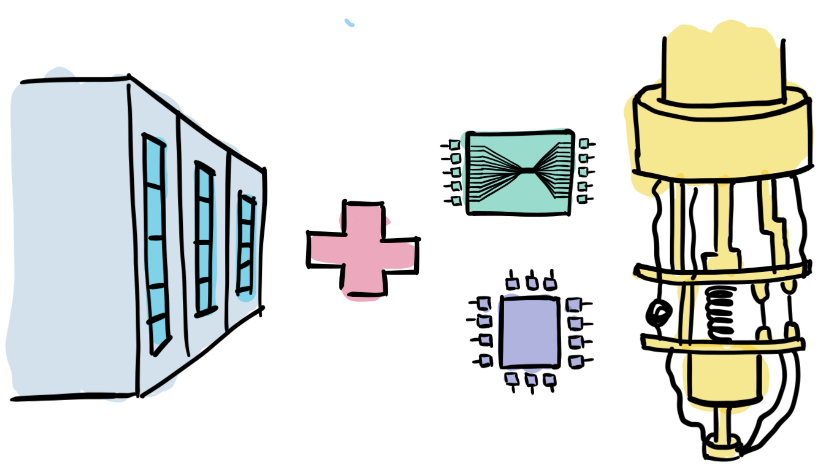


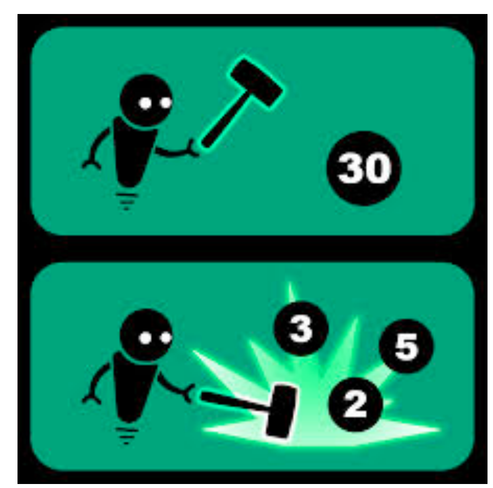
- Simulating Quantum Systems
- Optimization
- Cryptography

  Pirandola, et al, Adv. Opt. Photon. (2020)
- Machine Learning

  Biamonte, et al, Nature (2017)

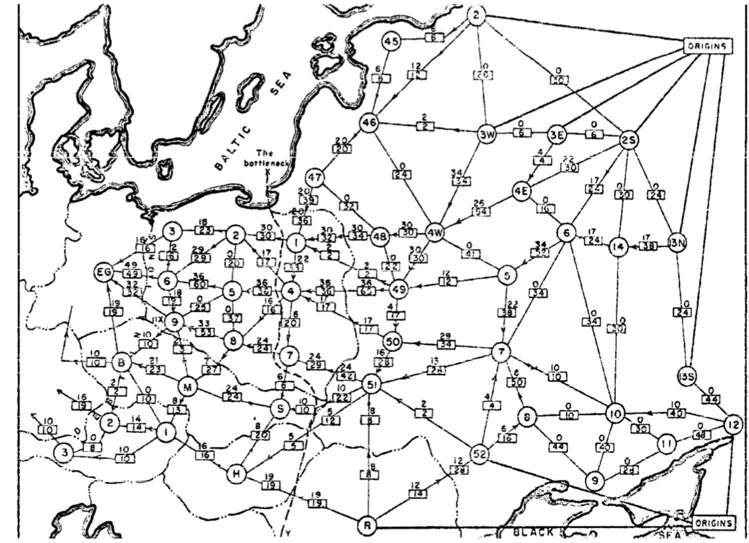






Which problems can be solved by a quantum computer?

- **Simulating Quantum Systems**
- Optimization



- Cryptography

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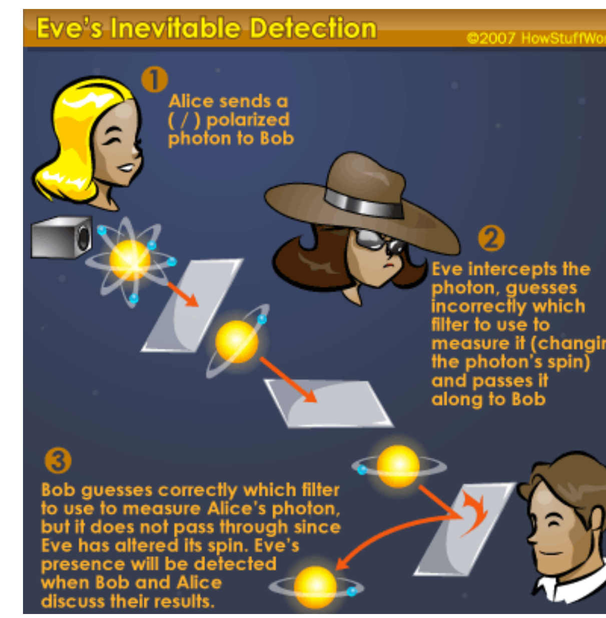

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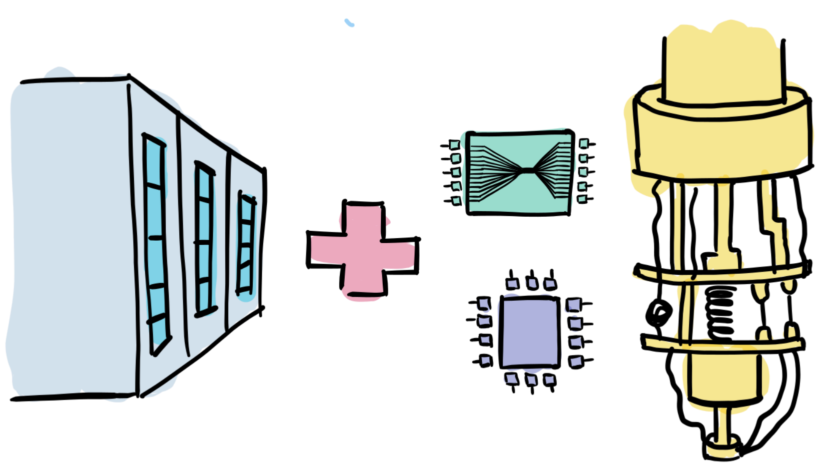
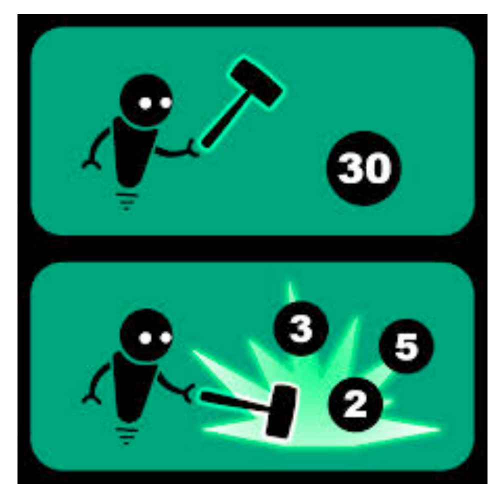




The first idea for quantum computing

"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

Richard Feynman (1982)

Simulating quantum systems

Feynman proved to be correct

Universal Quantum Simulators

Seth Lloyd

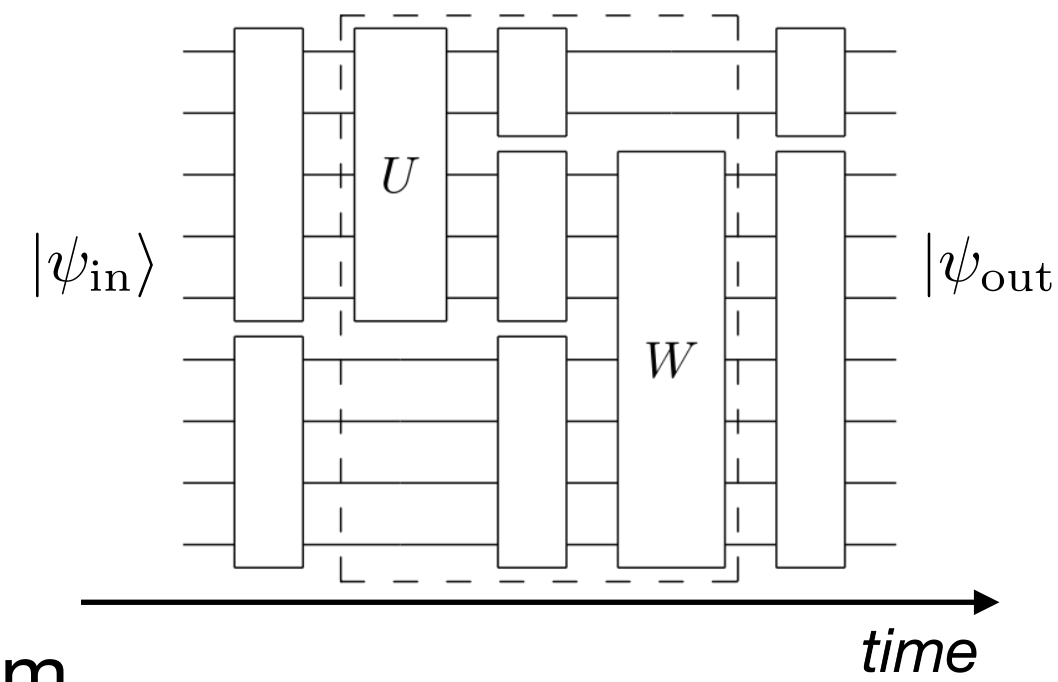
Feynman's 1982 conjecture, that quantum computers can be programmed to simulate any local quantum system, is shown to be correct.

Here I show that a quantum computer can in fact simulate quantum systems efficiently as long as they evolve according to local interactions.

Many applications…

Materials science, Complex systems, Physics, Chemistry, Biomedicine (protein folding/dynamics),

Simulating quantum systems



How to perform such simulations?

Two big families:

Hamiltonian simulations: dynamics
Design the evolution of the quantum system

- Not quite feasible on NISQ devices

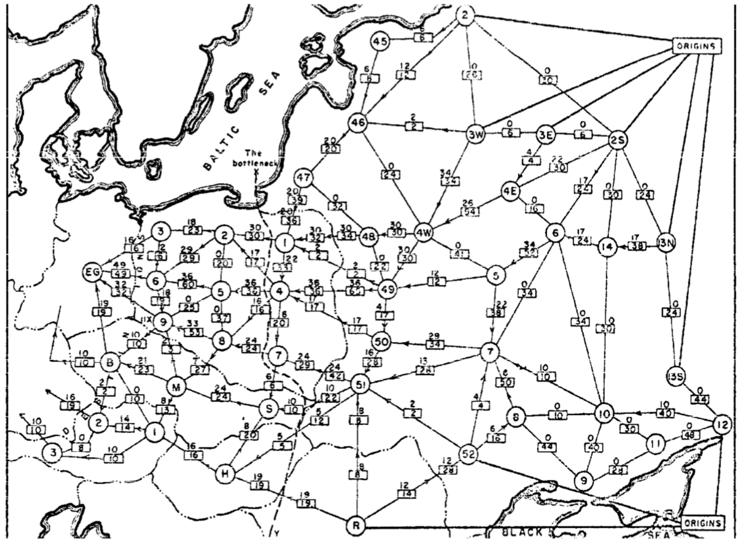
Variational Quantum Eigensolvers: groundstates
Simulate only the low-energy state of the quantum system

Peruzzo, et al. A variational eigenvalue solver on a photonic quantum processor. Nat Commun (2014)

- In reach of NISQ

Which problems can be solved by a quantum computer?

- Simulating Quantum Systems
- **Optimization**



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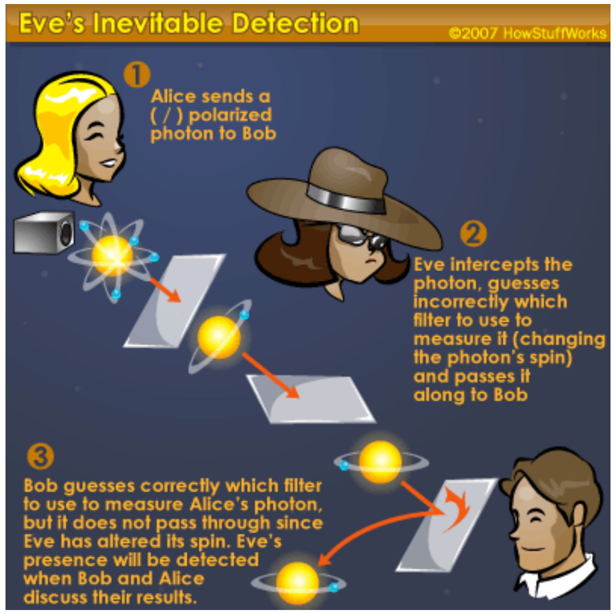

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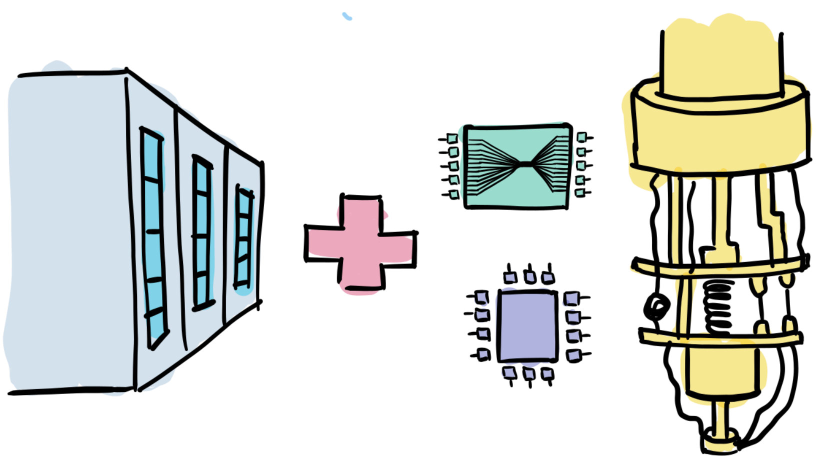
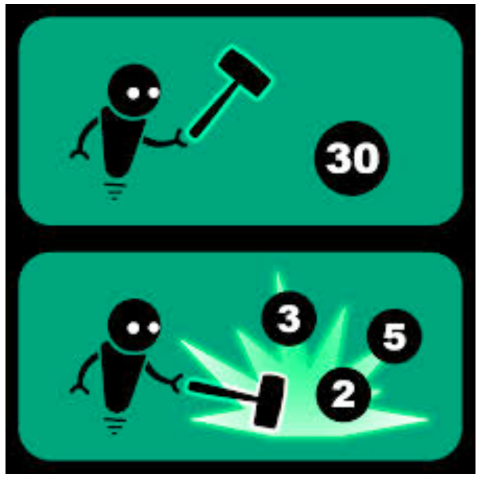


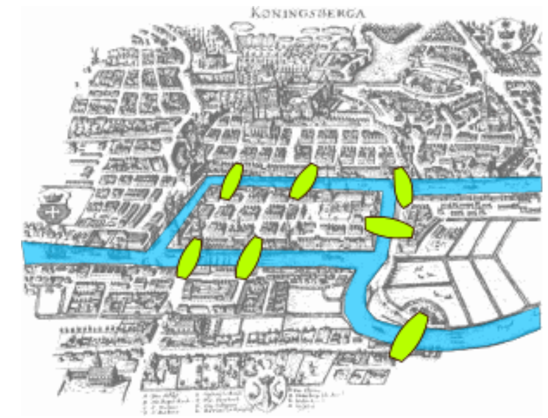


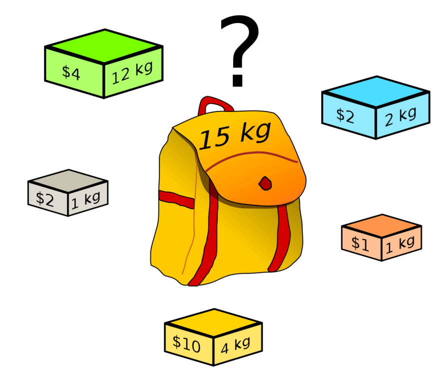
Optimization

✦ Optimization problems are ubiquitous in every-day life

- Cost function to be minimized/maximized
- (Often) constraints

Traveling salesman problem
Knapsack problem





✦ Applications in engineering, economics, data analysis, industry…

✦ Theoretical computer science

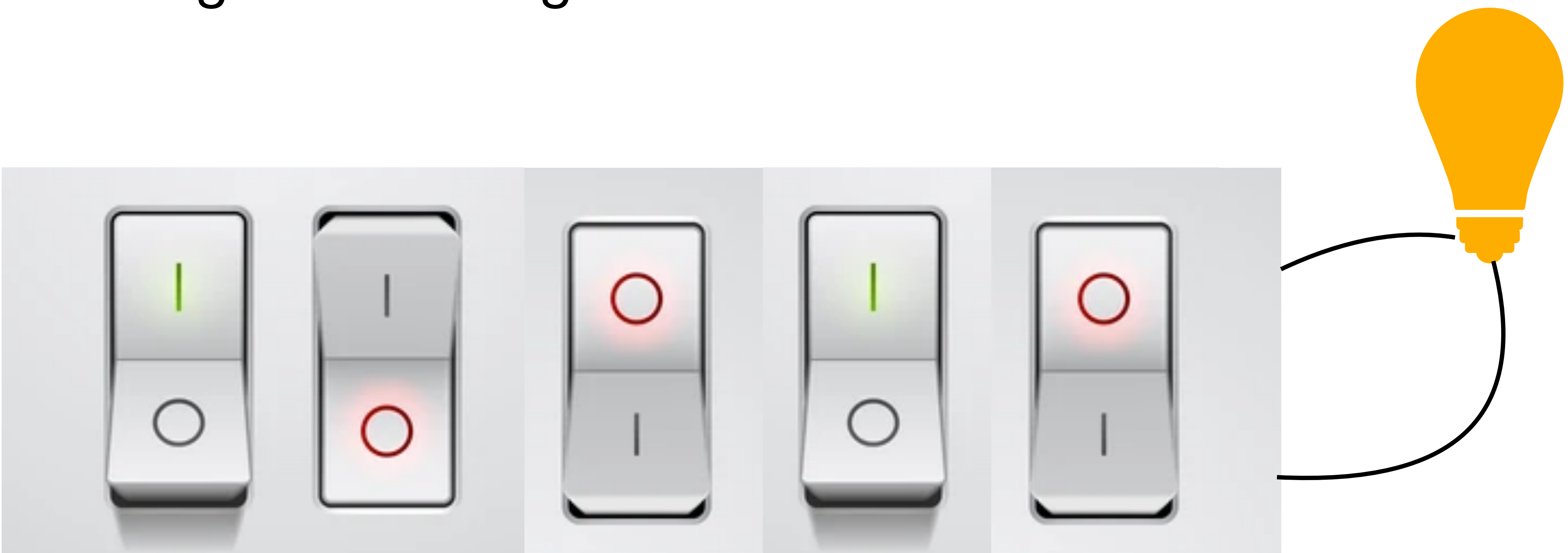
Define degree of difficulty (complexity classes P, NP, …)

A "difficult" problem requires exp(*N*) steps in the input size *N* to get to the correct answer

How can quantum computers help?

- Using quantum algorithms to mitigate the exp(N) scaling

  e.g. Grover's algorithm



Computing the right configuration that turns the bulb on requires 2^N steps

- **Grover's algorithm** reduces the number of steps to $2^{N/2}$ steps

… not very feasible on NISQ

What can we do on current NISQ devices?

A Quantum Approximate Optimization Algorithm

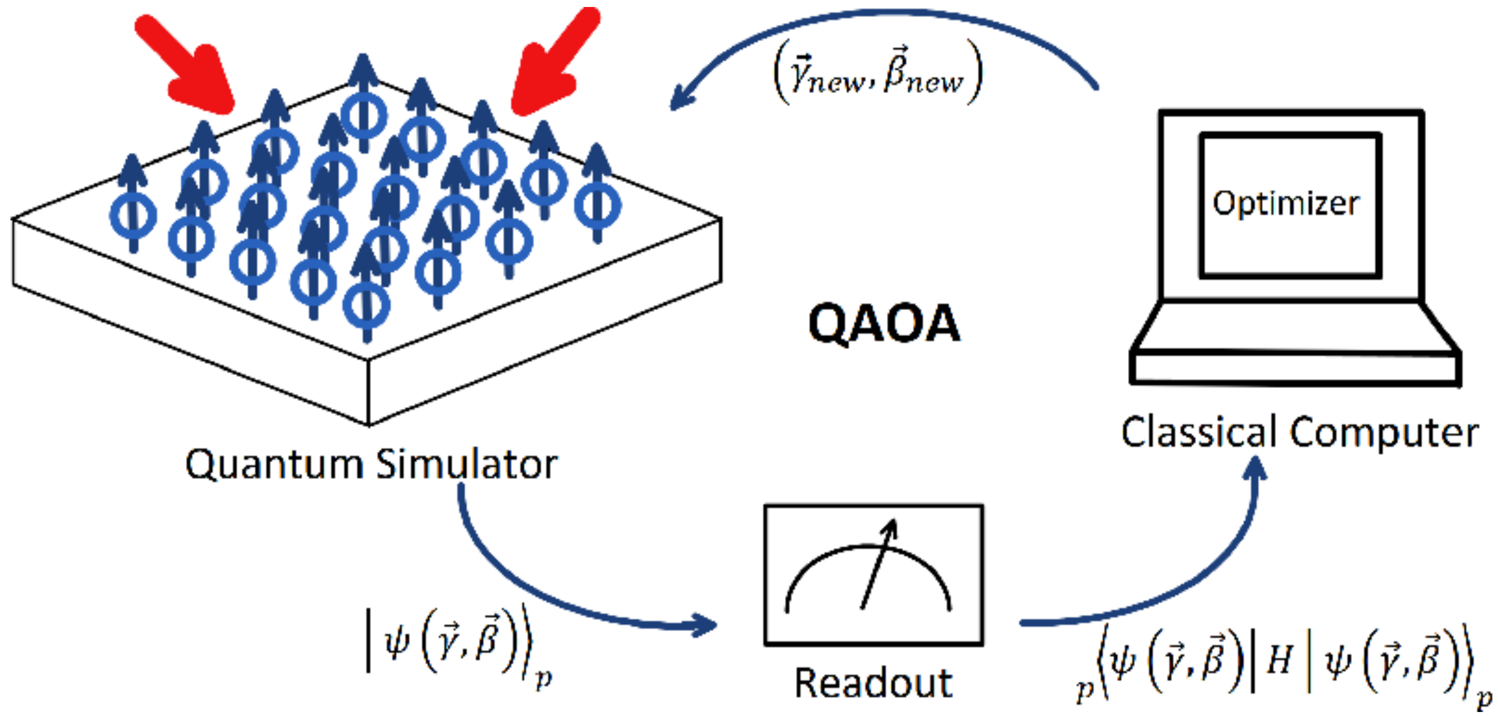
Edward Farhi and Jeffrey Goldstone
Center for Theoretical Physics
Massachusetts Institute of Technology
Cambridge, MA 02139

Sam Gutmann

- Belongs to the family of Variational Quantum Algorithms
- NISQ feasible but we don't know the amount of speedup

Fahri et al, arxiv:1411.4028

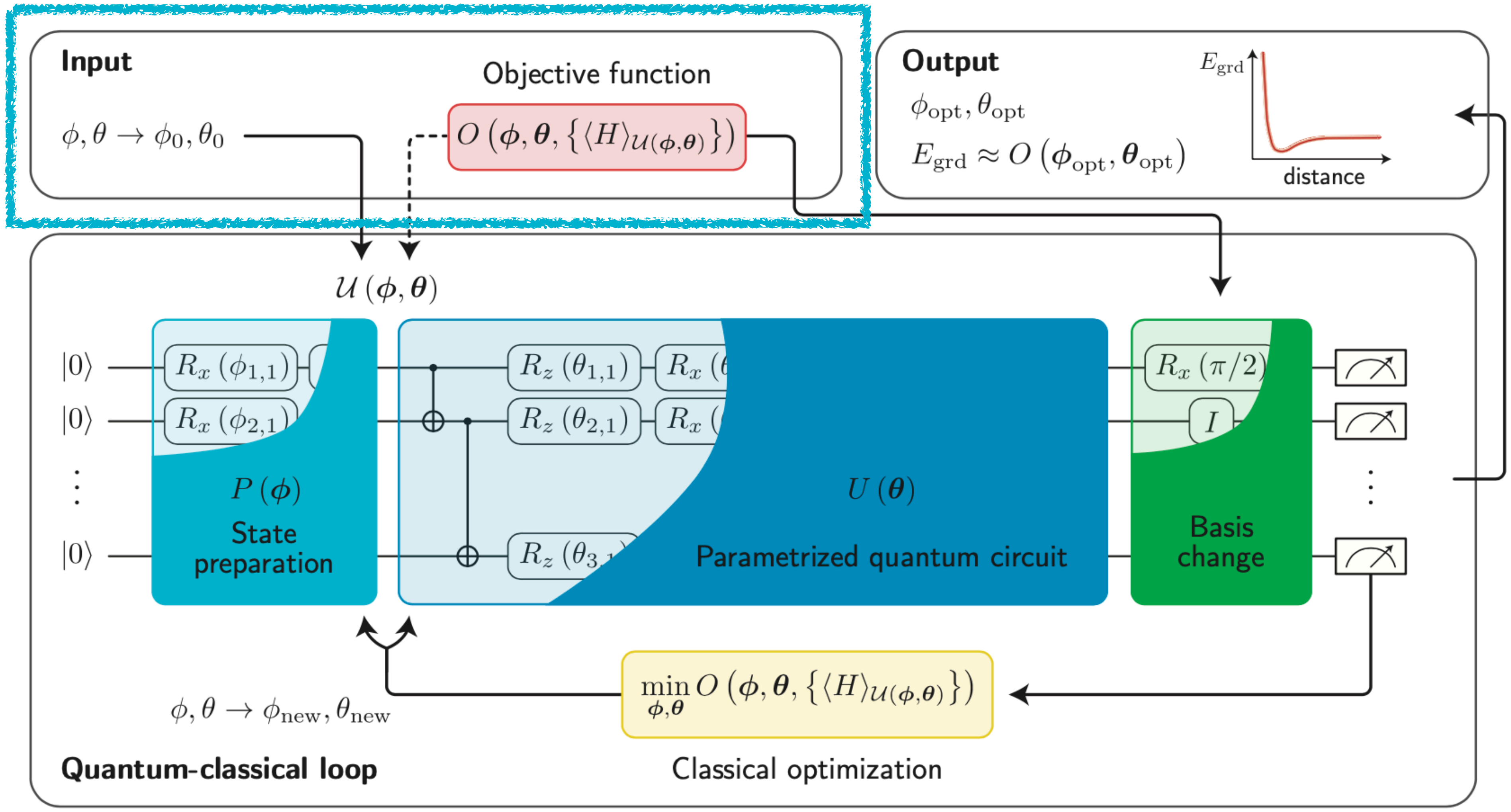
QAOA



Goal:

Minimize an objective/cost function

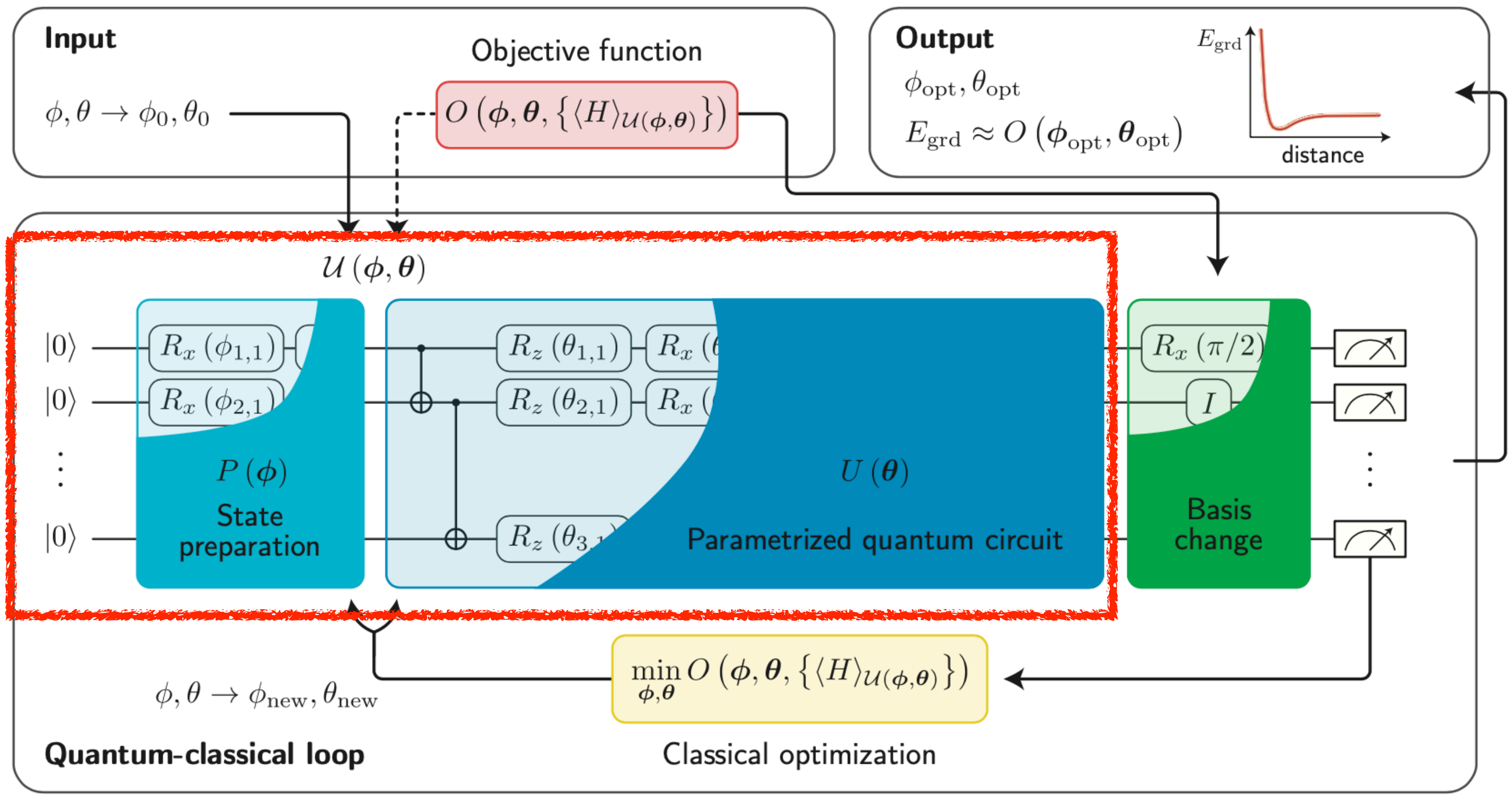
- Hybrid algoritm
  - The problem is encoded in a quantum state
  - A quantum evolution is performed via a parametric quantum circuit
  - The final state is measured and its cost is computed
  - The parameters are updated via a classical optimizer



Objective (cost) function

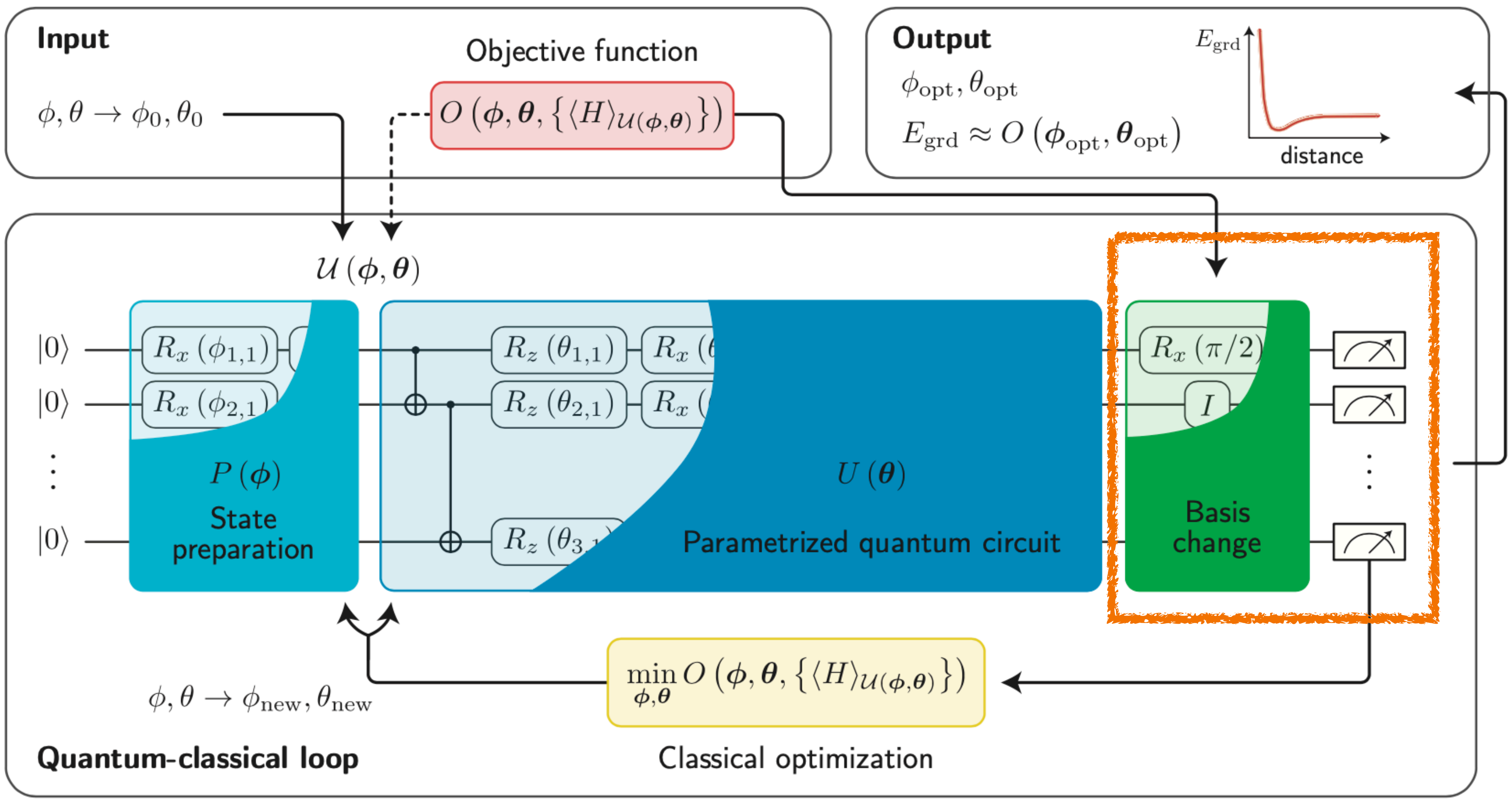
- embedding of degrees of freedom in Hilbert space
- encoding of the cost function in some quantum Hamiltonian *H*
- The solution is the ground state of *H*

QAOA



- Quantum circuit

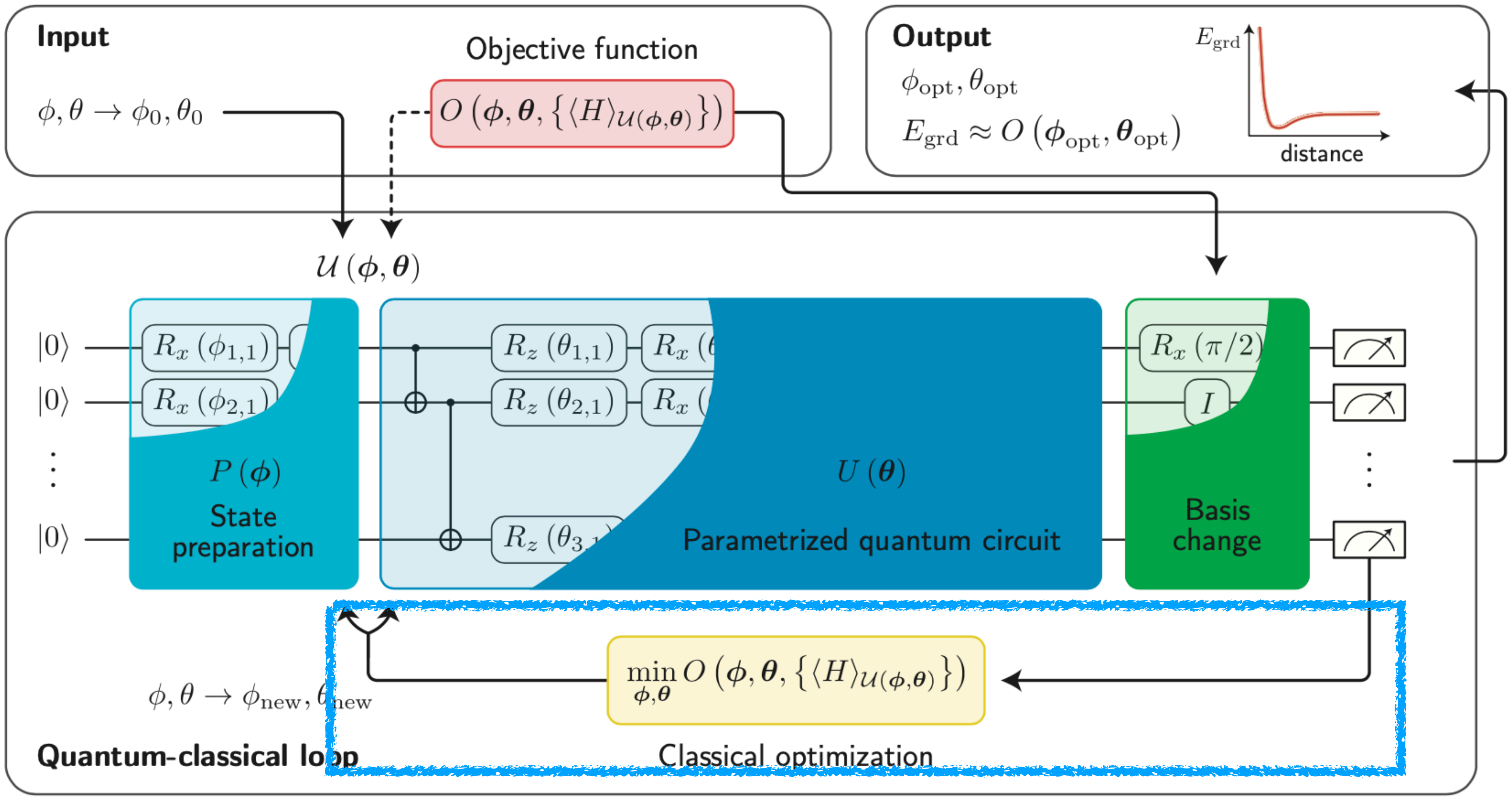
- preparation of initial state
- unitary evolution to span the space of possible ground states



- Measurement scheme

- suitable changes of bases
- measurements to obtain $\langle H \rangle_{U(\phi,\theta)}$

QAOA

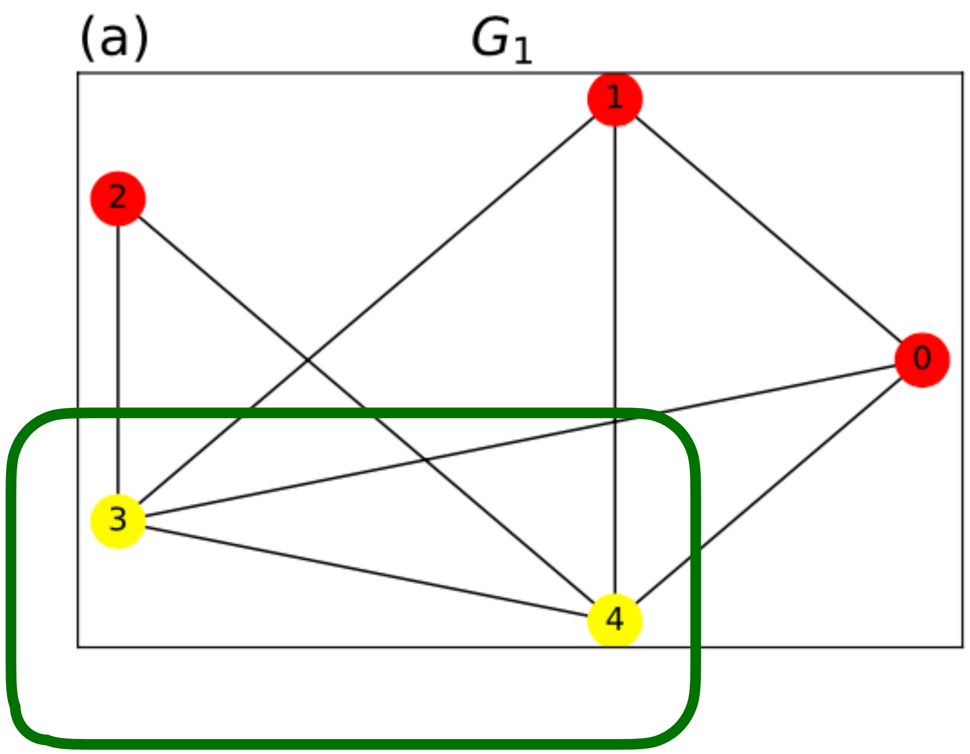


- Optimization

- based on classical algorithms (e.g. gradient based approaches, …)
- Bayesian adaptive techniques

MAX-CUT problem

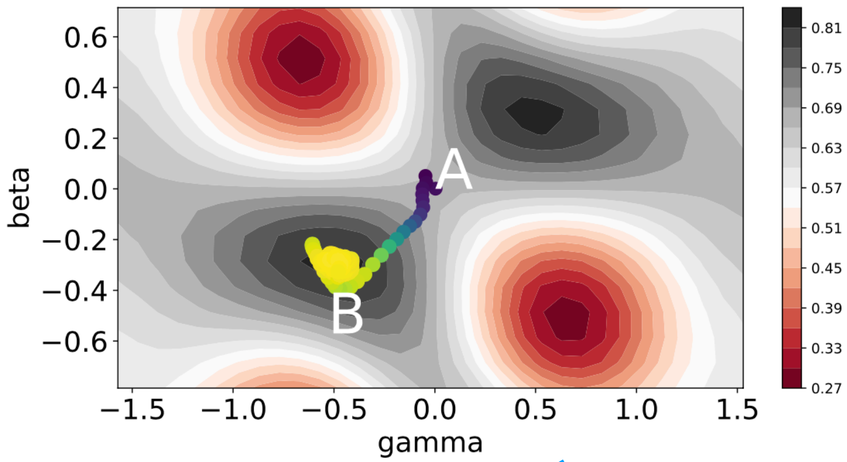
Find a partition of the graph's vertices into two complementary sets S and T, such that the number of edges between the set S and the set T is as large as possible.

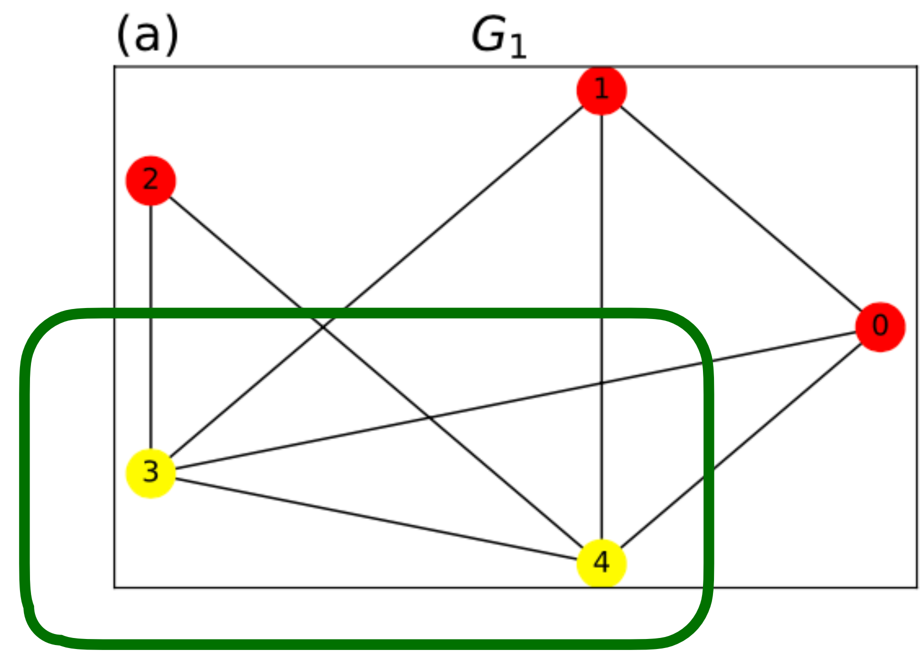


QAOA: an application

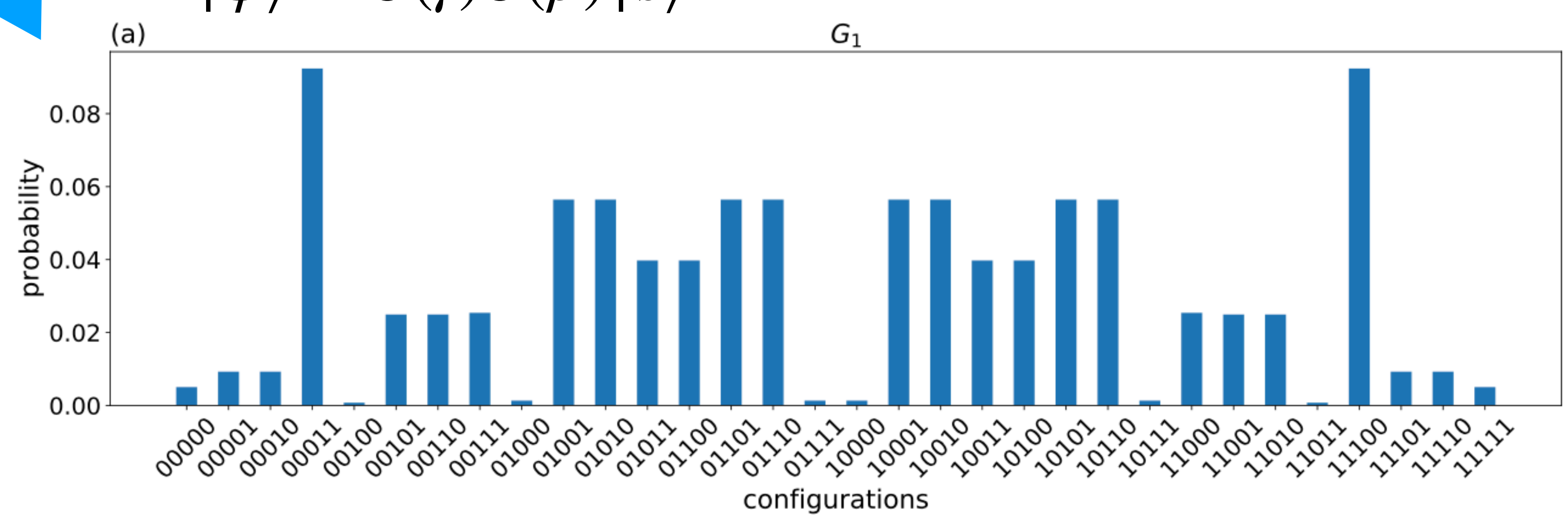
MAX-CUT problem

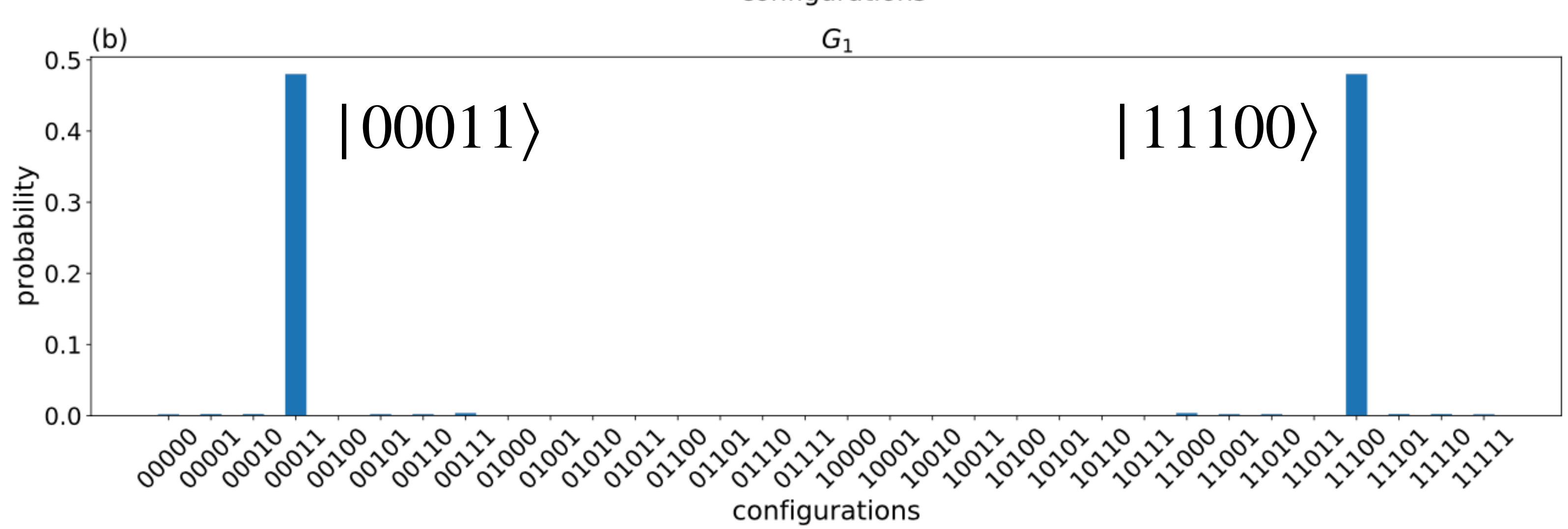
$$|s\rangle = |00000\rangle + |00001\rangle + \ldots + |11111\rangle$$





$$|\phi\rangle = U(\gamma)U(\beta)\,|s\rangle$$





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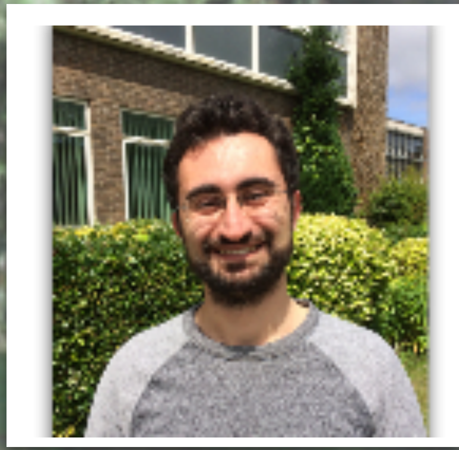

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Cristian Degli Esposti Boschi (CNR)


Thank you!